Exploring the Statistics of Radioactive Decay

Paul Secular[1], Imperial College London

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Abstract

The statistics of the radioactive decay of a strontium-90 source were investigated using a silicon detector and amplifier connected to a computer. The random nature of the decay was found to be compatible with the theoretical statistical models. In particular, the distribution of the number of counts and the distribution of the time between counts were found to be consistent with the Poisson and exponential distributions, respectively.

I. Introduction

The statistical nature of radioactivity was investigated thoroughly by Rutherford and Soddy at the beginning of the 20th century, leading to the publication of their empirically derived exponential decay law [1]. It was later realised that the randomness of radioactive decay arises from the probabilistic laws of quantum mechanics which govern behaviour at the atomic scale [2]. Interestingly, theoretical calculations actually suggest that the exponential decay law does not hold over extremely short or extremely long timescales, but that it should generally be obeyed for the observable lifespan of radioactive samples [3].

The present work describes a simple undergraduate-level experiment which was undertaken to examine the statistics of radioactive decay. An investigation of this type serves an important pedagogical purpose to the undergraduate of physics for two reasons:

Firstly, it provides an excellent way of concreting the notoriously difficult concepts of statistics and probability theory in the student's mind. For example, the vast numbers of nuclei present in a macroscopic quantity of radioactive material allow for direct observation of the law of large numbers in action.

Secondly, it illustrates the striking reality of the quantum world, whose wavefunctions and probability amplitudes seem so far removed from the determinism of our everyday classical experiences.

II. Methodology

The experiment consisted of a low-activity beta emitter and a silicon wafer detector held in place by retort stands and placed a small distance apart. The detector was connected to an amplifier which was in turn connected to a computer via a National Instruments Data Acquisition Board. The LabVIEW software package was used to collate and display data from the detector in real-time.

Strontium-90 is readily available in university laboratories and was chosen as the source for this experiment because of its long half-life (28.8 years [4]). This means its activity remains effectively constant during the timescale of the experiment (i.e. two to three hours).

In order to vary the average number of counts detected, a number of thin aluminium sheets were placed on top of the detector to shield it from the source. For more detailed information on the apparatus employed, see [5]

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[1] paul@secular.me.uk

III. The number of counts in a time interval

The possible decay of a single nucleus within a given time interval can be viewed as a Bernoulli trial. In other words, the nucleus either decays or doesn't decay, randomly, and independently of any external factors. Therefore the total number of nuclei decaying from a sample during a certain time interval is properly described by the binomial distribution.

However, working with the binomial distribution requires knowledge of the exact number of trials. That is to say the number of nuclei needs to be known. In practice, any radioactive material will contain a vast quantity of atoms whose number are reducing exponentially as they decay. If such a macroscopic sample is considered and a time interval is chosen that is very small compared to its half-life, then the number of decays occurring during that interval will also be very small relative to the total number of radioactive nuclei.

In this instance, the Poisson distribution can be used to approximate the binomial. This is because the Poisson is actually the mathematical limit of the binomial where the number of trials approaches infinity and the probability of an event approaches zero. This approximation is known as the law of rare events [6].

The product of the number of "trials" (nuclei) and the probability of one "success" (decay) gives the mean number of "successes" (decays) expected to occur in the chosen time interval. If this were to hold true for all time, then the binomial or Poisson distribution would describe the counting statistics for an interval chosen arbitrarily and hold true for repeated measurements. However, this would require all decayed nuclei to somehow be replaced. In practice therefore, this mean value is not constant and so the distribution describing the count will vary with time. If measurements are made during a window that is far smaller than the radionuclide's half-life though, then this mean value will change very little and can be approximated as constant.

The strontium-90 source used in this experiment has a very long half-life, so the rate of decay during one day, or even one month, remains approximately constant and the Poisson distribution can be used to describe its behaviour.

Now, both the binomial and Poisson distributions are discrete. Physically this corresponds to the atomic, rather than continuous, structure of matter. However, in the limit of a large mean, the Poisson can be approximated by the continuous Gaussian distribution. Just as the Poisson is more practical than the binomial, the Gaussian is itself often more useful than the Poisson due to it simplifying most statistical formulae [7].

To test these statistical models, repeated counts were taken from the strontium-90 source. The data measured for a small mean count rate were compared to a Poisson distribution (figure 1). Six of the ten data points were found to be in agreement to within one standard error. This is slightly lower than might be expected but suggests a plausible fit.

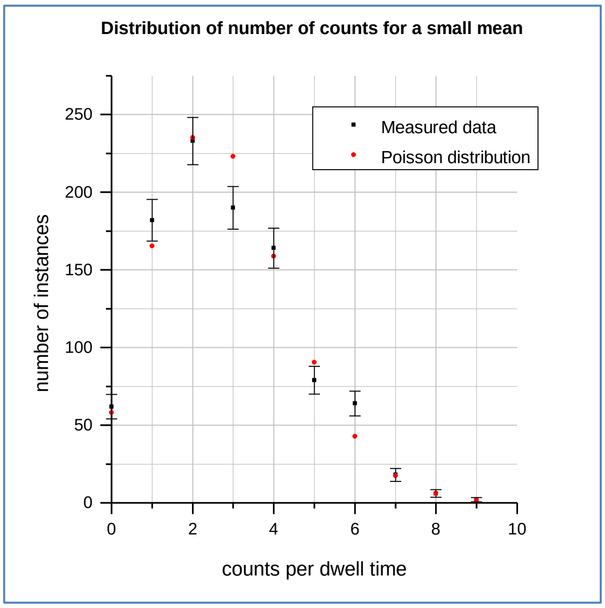

Figure 1. Experimental data showing number of times that a particular count was detected during a dwell time of 0.10s. The mean count was 2.846. 1000 total measurements were taken. Theoretical (Poisson) values also shown in red.

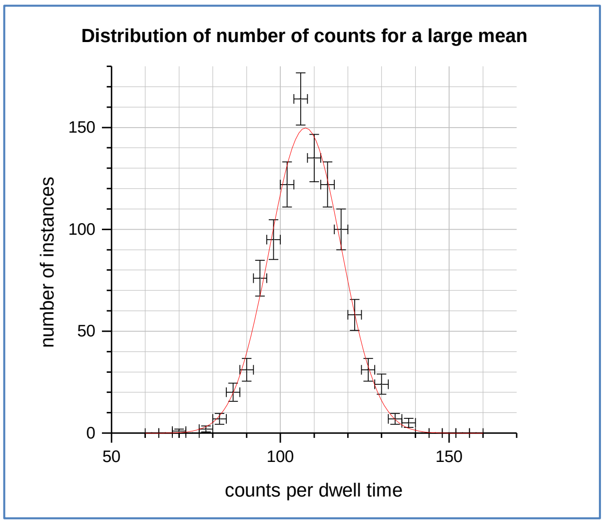

Figure 2. Experimental data showing the number of times that a particular count was detected during a dwell time of 0.10s. The mean count was 107.403. 1000 total measurements were taken. Theoretical (Gaussian) curve also shown for comparison.

The data measured for a large mean count rate were compared to a continuous Gaussian distribution (figure 2). In this instance, 96% of the measured data points can be seen to match the theoretical curve to within one standard error. Although this is a better fit than expected, it clearly shows that the data are compatible with the theory.

It should be mentioned that the "errors" mentioned above are not actually due to any experimental uncertainty but represent the inherent statistical variation from the expectation values due to the random nature of counting experiments.

IV. The time between counts

Under the conditions discussed above, for which the Poisson distribution holds, the process of repeated decays can be modelled as a Poisson process [2]. The assumption here is that no two decay events occur simultaneously. This is a reasonable assumption when the probability of a decay occurring during some time interval is very small.

It is trivial to show that if radioactive decay can be considered a Poisson process, then the probability distribution for the time of decay of a single nucleus is exponential. From this, the Rutherford-Soddy decay law follows.

It can also be shown that, for Poisson processes, the time between events follows an exponential distribution for any non-zero time-interval (by definition, there is zero probability of a zero inter-event time).

To test this model, the timing functionality of the software was used in order to measure the times between decays. The data are shown in figure 3.

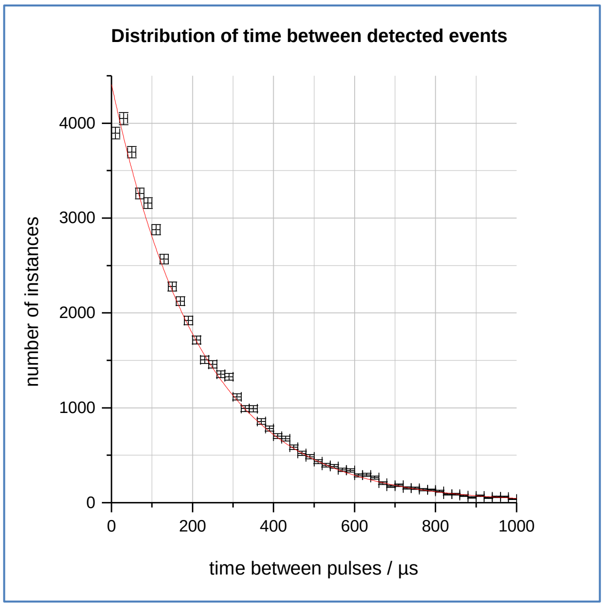

Figure 3. Experimental data showing the distribution of times between pulses picked up by the silicon detector. Mean rate was approximately 4550 counts per second. Theoretical exponential curve also shown for comparison.

The first thing to note about the graph is that there is an outlier at 10µs. This was due to the so-called "dead time" of the detector and was thus disregarded in subsequent analysis. Any detector will necessarily have a finite minimum response time after it detects a pulse, meaning that multiple events cannot be distinguished during this interval. This is known as the dead time and must always be taken into account.

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[2] Technically a homogeneous Poisson process. A Poisson process whose mean varies with time is referred to as nonhomogeneous.

To check the correlation between the data and the exponential curve, the data was replotted with a logarithmic vertical axis. A standard linear fit was computed using OriginPro, which gave an adjusted R^2 of 0.998. A value close to 1 implies a good fit, so this clearly suggests that the data are in excellent agreement with the model.

V. Conclusion

The statistical nature of radioactive decay has been discussed at some length leading to an explanation for the Rutherford-Soddy decay law. The assumptions and approximations inherent in the statistical models have also been noted.

It has been shown that an undergraduate-level experiment with the appropriate computer hardware and software can be used to easily check the probabilistic models describing the number of counts per time interval and the time between individual events. For the case of the latter distribution, it was noted that the dead time of a detector must be considered in order to explain the discrepancy from the theoretical exponential curve.

References

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